Section A: Q.1 – Q.10 Carry ONE mark each.

Q.1 The total number of Na and Cl ions per unit cell of the NaCl crystal is:

(A) 2

(B) 4

(C) 8

(D) 16

Q.2 The sum of three binary numbers, 10110.10, 11010.01, and 10101.11, in decimal system is:

(A) 70.75

(B) 70.25

(C) 70.50

(D) 69.50

Q.3 Which of the following matrices is Hermitian as well as unitary?

(A) $\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$

(B) $\begin{pmatrix} 0 & i \\ i & 0 \end{pmatrix}$

(C) $\begin{pmatrix} 1 & -i \\ i & 1 \end{pmatrix}$

(D) $\begin{pmatrix} 0 & 1+i \\ 1-i & 0 \end{pmatrix}$

Q.4 The divergence of a 3-dimensional vector $\frac{\hat{r}}{r^3}$ ($\hat{r}$ is the unit radial vector) is:

(A) $-\frac{1}{r^4}$

(B) Zero

(C) $\frac{1}{r^3}$

(D) $-\frac{3}{r^4}$

Q.5 The magnitudes of spin magnetic moments of electron, proton and neutron are μ_e, μ_p and μ_n, respectively. Then,

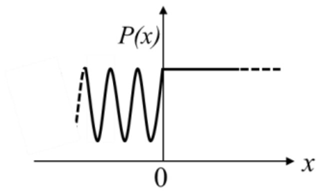
(A) $\mu_e > \mu_p > \mu_n$

(B) $\mu_e = \mu_p > \mu_n$

(C) $\mu_e < \mu_p < \mu_n$

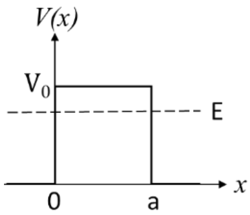
(D) $\mu_e < \mu_p = \mu_n$

Q.6 A particle moving along the x-axis approaches $x = 0$ from $x = -\infty$ with a total energy E. It is subjected to a potential $V(x)$. For time $t \to \infty$, the probability density $P(x)$ of the particle is schematically shown in the figure.

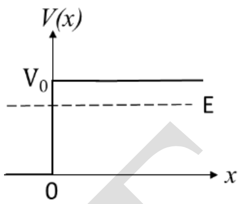


The correct option for the potential $V(x)$ is:

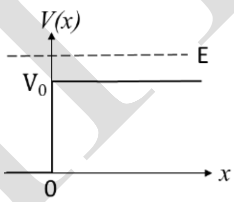
(A)



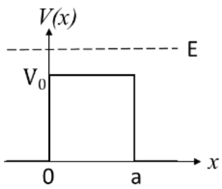
(B)



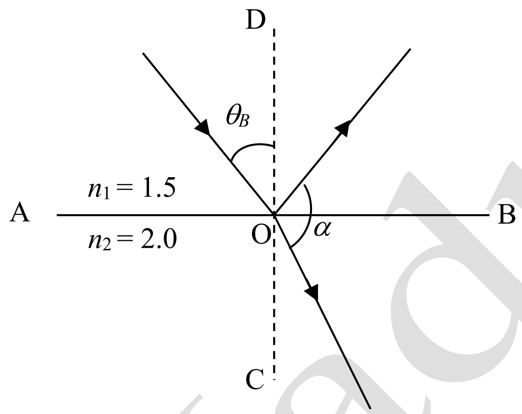
(C)



(D)



Q.7 A plane electromagnetic wave is incident on an interface AB separating two media (refractive indices $n_1 = 1.5$ and $n_2 = 2.0$) at Brewster angle θ_B, as schematically shown in the figure. The angle α (in degrees) between the reflected wave and the refracted wave is:



(A) 120

(B) 116

(C) 90

(D) 74

Q.8 If the electric field of an electromagnetic wave is given by,

$$\vec{E} = (4\hat{x} + 3\hat{y})e^{i(\omega t + ax - 600y)},$$

then the value of a is:

(all values are in the SI units)

(A) 450

(B) -450

(C) 800

(D) -800

Q.9 A vector field is expressed in the cylindrical coordinate system (s, ϕ, z) as,

$$\vec{F} = \frac{A}{s}\hat{s} + \frac{B}{s}\hat{z}.$$

If this field represents an electrostatic field, then the possible values of A and B, respectively, are:

(A) 1 and 0

(B) 0 and 1

(C) -1 and 1

(D) 1 and -1

Q.10 Which of the following types of motion may be represented by the trajectory, $y(x) = ax^2 + bx + c$?

(Here a, b, and c are constants; x, y are the position coordinates)

(A) Projectile motion in a uniform gravitational field

(B) Simple harmonic motion

(C) Uniform circular motion

(D) Motion on an inclined plane in a uniform gravitational field

Section A: Q.11 – Q.30 Carry TWO marks each.

Q.11 A crystal plane of a lattice intercepts the principal axes $\vec{a}_1, \vec{a}_2,$ and $\vec{a}_3$ at $3a_1, 4a_2,$ and $2a_3$, respectively. The Miller indices of the plane are:

(A) (436)

(B) (342)

(C) (634)

(D) (243)

Q.12 The number of atoms in the *basis* of a primitive cell of hexagonal closed packed structure is:

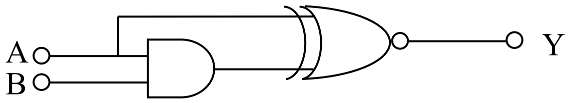
(A) 1

(B) 2

(C) 3

(D) 4

Q.13 Consider the following logic circuit.



The output Y is LOW when:

(A) *A* is HIGH and *B* is LOW

(B) *A* is LOW and *B* is HIGH

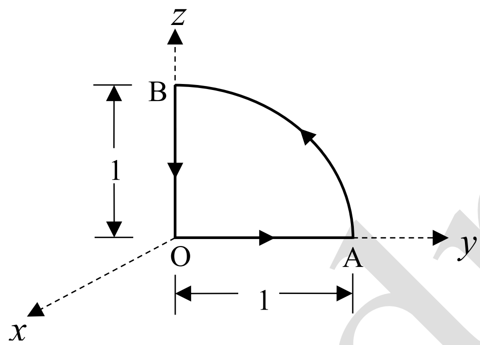
(C) Both *A* and *B* are LOW

(D) Both *A* and *B* are HIGH

Q.14 The value of the line integral for the vector,

$$\vec{v} = 2\hat{x} + yz^2\hat{y} + (3y + z^2)\hat{z}$$

along the closed path OABO (as shown in the figure) is:



(Path AB is the arc of a circle of unit radius)

(A) $\frac{1}{4}(3\pi - 1)$

(B) $3\pi - \frac{1}{4}$

(C) $\frac{3\pi}{4} - 1$

(D) $3\pi - 1$

Q.15 In the x-y plane, a vector is given by

$$\vec{F}(x,y) = \frac{-y\hat{x} + x\hat{y}}{x^2 + y^2}.$$

The magnitude of the flux of $\vec{\nabla} \times \vec{F}$, through a circular loop of radius 2, centered at the origin, is:

(A) π

(B) 2π

(C) 4π

(D) 0

Q.16 The roots of the polynomial, $f(z) = z^4 - 8z^3 + 27z^2 - 38z + 26,$ are $z_1, z_2, z_3, \& \, z_4$, where z is a complex variable. Which of the following statements is correct?

(A) $\dfrac{z_1 + z_2 + z_3 + z_4}{z_1 z_2 z_3 z_4} = -\dfrac{4}{19}$

(B) $\dfrac{z_1 + z_2 + z_3 + z_4}{z_1 z_2 z_3 z_4} = \dfrac{4}{13}$

(C) $\dfrac{z_1 z_2 z_3 z_4}{z_1 + z_2 + z_3 + z_4} = -\dfrac{26}{27}$

(D) $\dfrac{z_1 z_2 z_3 z_4}{z_1 + z_2 + z_3 + z_4} = \dfrac{13}{19}$

Q.17 The ultraviolet catastrophe in the classical (Rayleigh-Jeans) theory of cavity radiation is attributed to the assumption that

(A) the standing waves of all allowed frequencies in the cavity have the same average energy

(B) the density of the standing waves in the cavity is independent of the shape and size of the cavity

(C) the allowed frequencies of the standing waves inside the cavity have no upper limit

(D) the number of allowed frequencies for the standing waves in a frequency range ν to $(\nu + d\nu)$ is proportional to ν^2

Q.18 Given that the rest mass of electron is 0.511MeV/c^2, the speed (in units of c) of an electron with kinetic energy 5.11MeV is closest to:

(A) 0.996

(B) 0.993

(C) 0.990

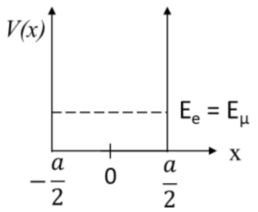
(D) 0.998

Q.19 A one-dimensional infinite square-well potential is given by:

$$V(x) = 0 \quad for -\frac{a}{2} < x < +\frac{a}{2}$$
$$= \infty \quad elsewhere$$

Let $E_e(x)$ and $\psi_e(x)$ be the ground state energy and the corresponding wave function, respectively, if an electron (e) is trapped in that well. Similarly, let $E_\mu(x)$ and $\psi_\mu(x)$ be the corresponding quantities, if a muon (μ) is trapped in the well. Choose the correct option:

(A)



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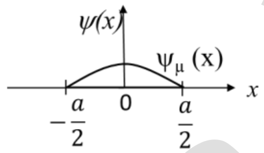
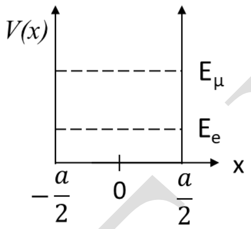


(B)



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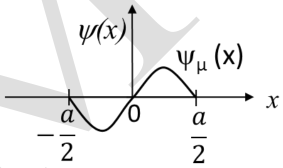
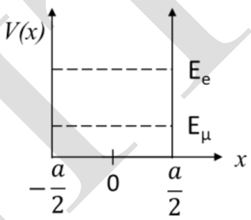


(C)



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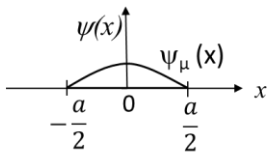
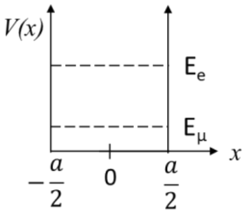


(D)



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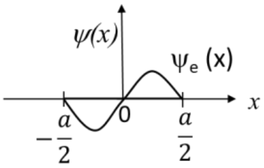
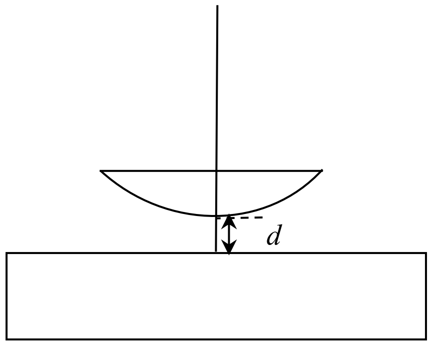


Q.20 In a Newton's rings experiment (using light of free space wavelength 580nm), there is an air gap of height d between the glass plate and a plano-convex lens (see figure). The central fringe is observed to be bright.



The least possible value of d (in nm) is:

(A) 145

(B) 290

(C) 580

(D) 72.5

Q.21 Linearly polarized light (free space wavelength λ_0= 600nm) is incident normally on a retarding plate ($n_e - n_o = 0.05$ at λ_0= 600nm). The emergent light is observed to be linearly polarized, irrespective of the angle between the direction of polarization and the optic axis of the plate. The minimum thickness (in μm) of the plate is:

(A) 6

(B) 3

(C) 2

(D) 1

Q.22 A 15.7mW laser beam has a diameter of 4mm. If the amplitude of the associated magnetic field is expressed as $\frac{A}{\sqrt{\varepsilon_0 c^3}}$, the value of *A* is:

(ε_0 is the free space permittivity and c is the speed of light)

(A) 50

(B) 35.4

(C) 100

(D) 70.8

Q.23 The plane $z = 0$ separates two linear dielectric media with relative permittivities $\varepsilon_{r1} = 4$ and $\varepsilon_{r2} = 3$, respectively. There is no free charge at the interface. If the electric field in the medium 1 is $\vec{E}_1 = 3\hat{x} + 2\hat{y} + 4\hat{z}$, then the displacement vector $\vec{D}_2$ in medium 2 is:

(ε_0 is the permittivity of free space)

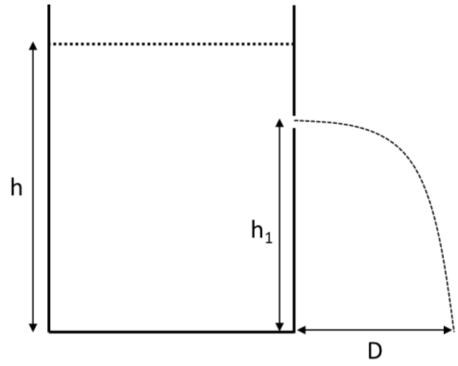
(A) $(3\hat{x} + 4\hat{y} + 6\hat{z})\varepsilon_0$

(B) $(3\hat{x} + 6\hat{y} + 8\hat{z})\varepsilon_0$

(C) $(9\hat{x} + 6\hat{y} + 16\hat{z})\varepsilon_0$

(D) $(4\hat{x} + 2\hat{y} + 3\hat{z})\varepsilon_0$

Q.24 A tank, placed on the ground, is filled with water up to a height h. A small hole is made at a height h_1 such that $h_1 < h$. The water jet emerging from the hole strikes



the ground at a horizontal distance *D*, as shown schematically in the figure. Which of the following statements is correct?

(g is the acceleration due to gravity)

(A) Velocity at h_1 is $\sqrt{2gh_1}$

(B) $D = 2(h - h_1)$

(C) *D* will be maximum when $h_1 = \frac{2}{3}h$

(D) The maximum value of *D* is *h*

Q.25 An incompressible fluid is flowing through a vertical pipe (height h and cross-sectional area A_o). A thin mesh, having n circular holes of area A_h, is fixed at the bottom end of the pipe. The speed of the fluid entering the top-end of the pipe is v_o. The volume flow rate from an individual hole of the mesh is given by:

(g is the acceleration due to gravity)

(A) $\frac{A_o}{n}\sqrt{v_o^2 + 2gh}$

(B) $\frac{A_o}{n}\sqrt{v_o^2 + gh}$

(C) $n(A_o - A_h)\sqrt{v_o^2 + 2gh}$

(D) $n(A_o - A_h)\sqrt{v_o^2 + gh}_o$

Q.26 A ball is dropped from a height h to the ground. If the coefficient of restitution is e, the time required for the ball to stop bouncing is proportional to:

(A) $\frac{2+e}{1-e}$

(B) $\frac{1+e}{1-e}$

(C) $\frac{1-e}{1+e}$

(D) $\frac{2-e}{1+e}$

Q.27 A cylinder-piston system contains *N* atoms of an ideal gas. If t_{avg} is the average time between successive collisions of a given atom with other atoms. If the temperature *T* of the gas is increased isobarically, then t_{avg} is proportional to :

(A) $\sqrt{T}$

(B) $\frac{1}{\sqrt{T}}$

(C) T

(D) $\frac{1}{T}$

Q.28 A gas consists of particles, each having three translational and three rotational degrees of freedom. The ratio of specific heats, C_p/C_v, is:

(C_p and C_v are the specific heats at constant pressure and constant volume, respectively)

(A) 5/3

(B) 7/5

(C) 4/3

(D) 3/2

Q.29 If two traveling waves, given by
$y_1 = A_0 \sin(kx - \omega t)$ and $y_2 = A_0 \sin(\alpha kx - \beta\omega t)$

are superposed, which of the following statements is correct?

(A) For $\alpha = \beta = 1$, the resultant wave is a standing wave

(B) For $\alpha = \beta = -1$, the resultant wave is a standing wave

(C) For $\alpha = \beta = 2$, the carrier frequency of the resultant wave is $\frac{3}{2}\omega$

(D) For $\alpha = \beta = 2$, the carrier frequency of the resultant wave is 3ω

Q.30 Suppose that there is a dispersive medium whose refractive index depends on the wavelength as given by $n(\lambda) = n_0 + \frac{a}{\lambda^2} - \frac{b}{\lambda^4}$. The value of λ at which the group and phase velocities would be the same, is:

(A) $\sqrt{\frac{2b}{a}}$

(B) $\sqrt{\frac{b}{2a}}$

(C) $\sqrt{\frac{3b}{a}}$

(D) $\sqrt{\frac{b}{3a}}$

Section B: Q.31 – Q.40 Carry TWO marks each.

Q.31 A pure Si crystal can be converted to an *n*-type crystal by doping with

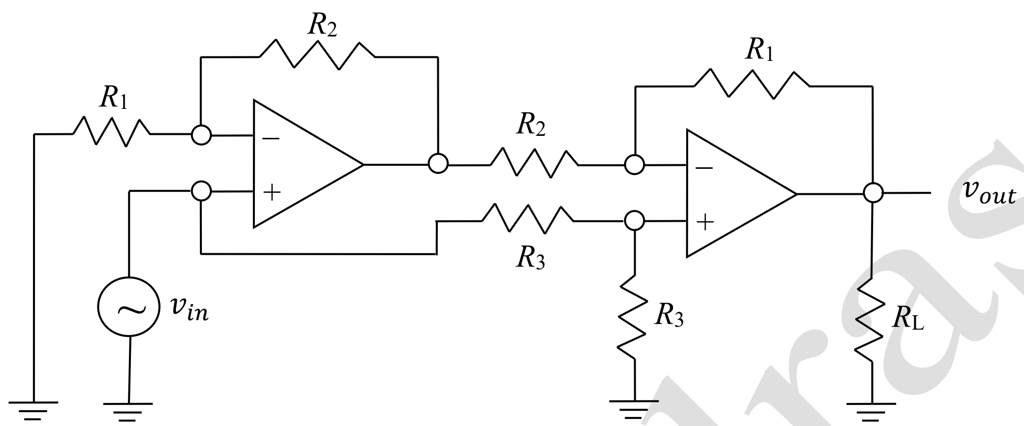
(A) P

(B) As

(C) Sb

(D) In

Q.32 In the following OP-AMP circuit, v_{in} and v_{out} represent the input and output signals, respectively.



Choose the correct statement(s):

(A) v_{out} is out-of-phase with v_{in}

(B) Gain is unity when $R_1 = R_2$

(C) v_{out} is in-phase with v_{in}

(D) v_{out} is zero

Q.33 A spring-mass system (spring constant 80N/m and damping coefficient 40N-s/m), initially at rest, is lying along the y-axis in the horizontal plane. One end of the spring is fixed and the mass (5kg) is attached at its other end. The mass is pulled along the y-axis by 0.5m from its equilibrium position and then released. Choose the correct statement(s).

(Ignore mass of the spring)

(A) Motion will be under damped

(B) Trajectory of the mass will be $y(t) = \frac{1}{2}(1 + t)e^{-4t}$

(C) Motion will be critically damped

(D) Trajectory of the mass will be $y(t) = \frac{1}{2}(1 + 4t)e^{-4t}$

Q.34 Consider two different Compton scattering experiments, in which X-rays and γ-rays of wavelength (λ) 1.024Å and 0.049Å, respectively, are scattered from stationary free electrons. The scattered wavelength (λ') is measured as a function of the scattering angle (θ). If Compton shift is $\Delta\lambda = \lambda' - \lambda$, then which of the following statement(s) is/are true:

($h = 6.63 \times 10^{-34}$ J.s, $m_e = 9.11 \times 10^{-31}$ kg, $c = 3 \times 10^8$ m/s)

(A) For γ-rays, $\lambda'_{\max} \approx 0.098$Å

(B) For X-rays, $(\Delta\lambda)_{\max}$ is observed at $\theta = 180^\circ$

(C) For X-rays, $(\Delta\lambda)_{\max} \approx 1.049$Å

(D) For γ-rays, at $\theta = 90^\circ$, $\lambda' \approx 0.049$Å

Q.35 A particle of mass m, having an energy E and angular momentum L, is in a parabolic trajectory around a planet of mass M. If the distance of the closest approach to the planet is r_{m}, which of the following statement(s) is(are) true?

(G is the Gravitational constant)

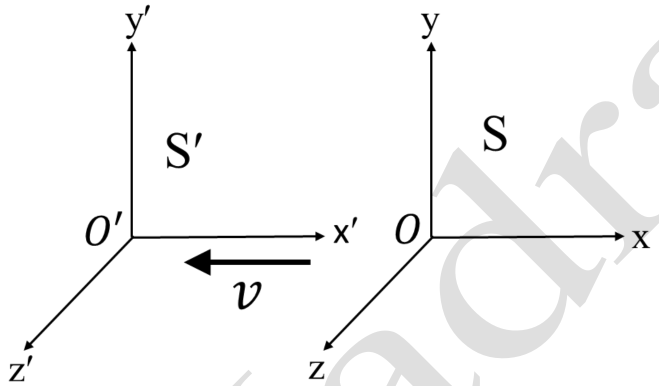
(A) $E > 0$

(B) $E = 0$

(C) $L = \sqrt{2GMm^2r_m}$

(D) $L = \sqrt{2GM^2mr_m}$

Q.36 The inertial frame S' is moving away from the inertial frame S with a speed $v = 0.6c$ along the negative x-direction (see figure). The origins O' and O of the frames coincide at $t = t' = 0$. As observed in the frame S', two events occur simultaneously at two points on the x'-axis with a separation of $\Delta x' = 5$m. If, Δt and Δx are the magnitudes of the time interval and the space interval, respectively, between the events in S, then which of the following statements is(are) correct?



($c = 3 \times 10^8$ m/s)

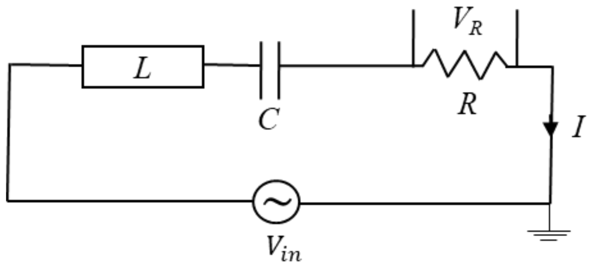
(A) $\Delta t = 12.5$ns

(B) $\Delta t = 4.2$ns

(C) $\Delta x = 10.6$m

(D) $\Delta x = 6.25$m

Q.37 For the LCR AC-circuit (resonance frequency ω_o) shown in the figure below, choose the correct statement(s).



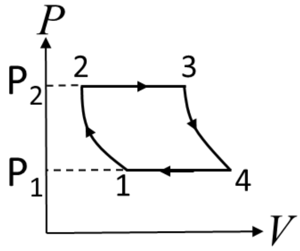
(A) ω_o depends on the values of L, C, and R

(B) At $\omega = \omega_o$, voltage V_R and current I are in-phase

(C) The amplitude of V_R at $\omega = \omega_o/2$ is independent of R

(D) The amplitude of V_R at $\omega = \omega_o$ is independent of L and C

Q.38 The P-V diagram of an engine is shown in the figure below. The temperatures at points 1, 2, 3 and 4 are T_1, T_2, T_3 and T_4, respectively. 1 $\rightarrow$2 and 3$\rightarrow$4 are adiabatic processes, and 2$\rightarrow$3 and 4$\rightarrow$1 are isochoric processes.



Identify the correct statement(s).

[γ is the ratio of specific heats C_p (at constant P) and C_v (at constant V)]

(A) $T_1T_3 = T_2T_4$

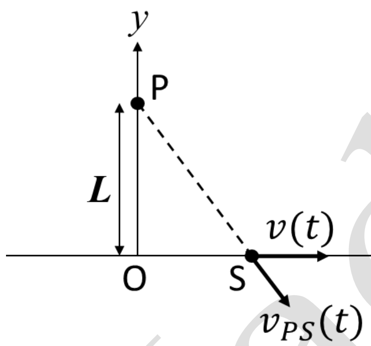
(B) The efficiency of the engine is $1 - \left(\frac{P_1}{P_2}\right)^{\frac{\gamma-1}{\gamma}}$

(C) The change in entropy for the entire cycle is zero

(D) $T_1T_2 = T_3T_4$

Q.39 A whistle S of sound frequency f is oscillating with angular frequency ω along the x-axis. Its instantaneous position and the velocity are given by $x(t) = a\sin(\omega t)$ and $v(t) = v_0 \cos(\omega t)$, respectively. An observer P is located on the y-axis at a distance L from the origin (see figure). Let $v_{PS}(t)$ be the component of $v(t)$ along the line joining the source and the observer. Choose the correct option(s):

(Here a and v_0 are constants)



(A) $v_{PS}(t) = \frac{1}{2}\frac{av_0}{\sqrt{a^2 sin^2 \omega t + L^2}} \sin(2\omega t)$

(B) The observed frequency will be f when the source is at $x = 0$ and $x = \pm a$

(C) The observed frequency will be f when the source is at position $x = \pm\frac{a}{2}$

(D) $v_{PS}(t) = \frac{1}{2}\frac{av_0}{\sqrt{a^2 + L^2}} \sin(2\omega t)$

Q.40 One mole of an ideal monoatomic gas, initially at temperature T_o is expanded from an initial volume V_o to $2.5V_o$. Which of the following statements is(are) correct?

(R is the ideal gas constant)

(A) When the process is isothermal, the work done is $RT_o \ln 2$

(B) When the process is isothermal, the change in internal energy is zero

(C) When the process is isobaric, the work done is $\frac{3}{2}RT_o$

(D) When the process is isobaric, the change in internal energy is $\frac{9}{2}RT_o$

Section C: Q.41 – Q.50 Carry ONE mark each.

Q.41 Consider a *p*-*n* junction diode which has 10^{23} acceptor-atoms/m^3 in the *p*-side and 10^{22} donor-atoms/m^3 in the *n*-side. If the depletion width in the *p*-side is 0.16μm, then the value of depletion width in the *n*-side will be ______μm. (Rounded off to one decimal place)

Q.42 The co-ordinate system (x,y,z) is transformed to the system (u,v,w), as given by:

$$u = 2x + 3y - z$$

$$v = x - 4y + z$$

$$w = x + y$$

The Jacobian of the above transformation is ____________.

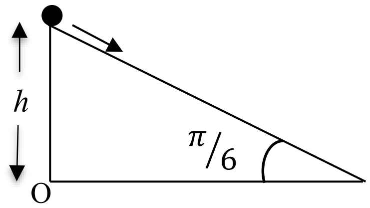
Q.43 Two sides of a triangle OAB are given by:

$$\overrightarrow{OA} = \hat{x} + 2\hat{y} + \hat{z}$$

$$\overrightarrow{OB} = 2\hat{x} - \hat{y} + 3\hat{z}$$

The area of the triangle is _________. (Rounded off to one decimal place)

Q.44 A particle of mass 1kg, initially at rest, starts sliding down from the top of a frictionless inclined plane of angle $\pi/6$ (as schematically shown in the figure). The magnitude of the torque on the particle about the point O after a time 2seconds is ___________N-m. (Rounded off to nearest integer)



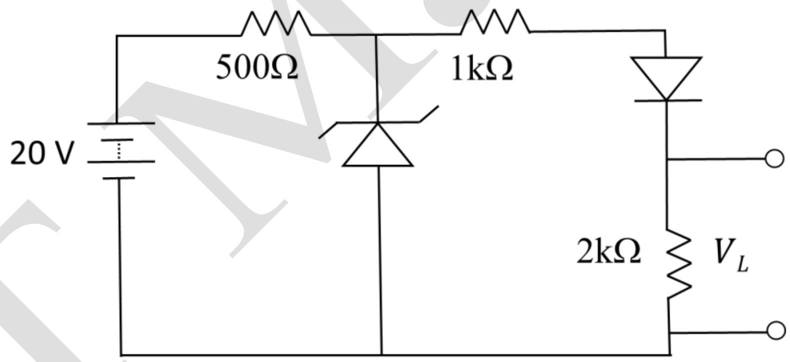
(Take g = 10m/s^2)

Q.45 The moment of inertia of a solid hemisphere (mass M and radius R) about the axis passing through the hemisphere and parallel to its flat surface is $\frac{2}{5}MR^2$. The distance of the axis from the center of mass of the hemisphere (in units of R) is________. (Rounded off to two decimal places)

Q.46 A collimated light beam of intensity I_0 is incident normally on an air-dielectric (refractive index 2.0) interface. The intensity of the reflected light is _______I_0. (Rounded off to two decimal places)

Q.47 A charge of -9C is placed at the center of a concentric spherical shell made of a linear dielectric material (relative permittivity 9) and having inner and outer radii of 0.1m and 0.2m, respectively. The total charge induced on its inner surface is ______C. (Rounded off to two decimal place)

Q.48 A Zener diode (rating 10V, 2W) and a normal diode (turn-on voltage 0.7V) are connected in a circuit as shown in the figure. The voltage drop V_L across the 2kΩ resistance is __________V. (Rounded off to one decimal place)



Q.49 The Fermi energy of a system is 5.5eV. At 500K, the energy of a level for which the probability of occupancy is 0.2, is ________eV. (Rounded off to two decimal places)

(Boltzmann constant $k_B = 8.62 \times 10^{-5}$ eV/K)

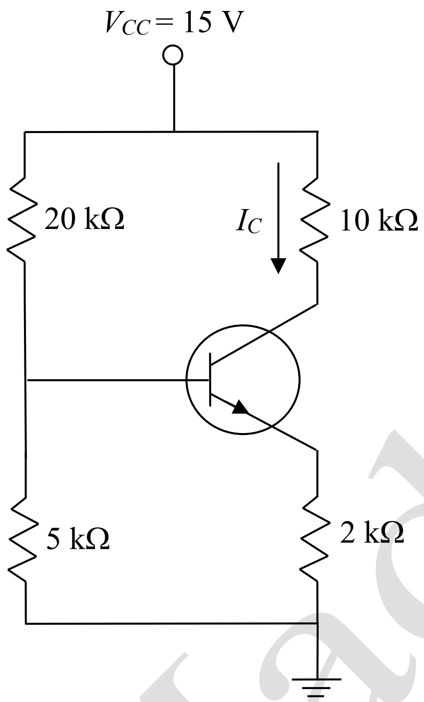
Q.50 One mole of an ideal monoatomic gas is heated in a closed container, first from 273K to 303K, and then from 303K to 373K. The net change in the entropy is ______R. (Rounded off to two decimal places)

(R is the ideal gas constant)

Section C: Q.51 – Q.60 Carry TWO marks each.

Q.51 For a simple cubic crystal, the smallest inter-planar spacing d that can be determined from its second order of diffraction using monochromatic X-rays of wavelength 1.32Å is _________Å. (Round off to two decimal places)

Q.52 A transistor (β=100, $V_{BE} = 0.7V$) is connected as shown in the circuit below.



The current I_C will be _________ mA. (Rounded off to two decimal places)

Q.53 In the Taylor expansion of function, $F(x) = e^x \sin x$, around $x = 0$, the coefficient of x^5 is ______________. (Rounded off to three decimal places)

Q.54 A stationary nitrogen $({}^{14}_{7}N)$ nucleus is bombarded with α - particle $({}^{4}_{2}He)$ and the following nuclear reaction takes place:

$$\begin{array}{lcccc} & {}^{4}_{2}He \;+ & {}^{14}_{7}N & \longrightarrow\; {}^{17}_{8}O \;+ & {}^{1}_{1}H \\ \text{Mass:} & 4.003u & 14.003u & 16.999u & 1.008u \end{array}$$

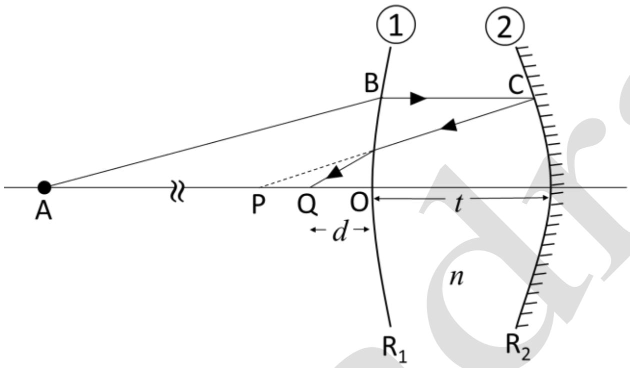
If the kinetic energies of ${}^{4}_{2}He$ and ${}^{1}_{1}H$ are 5.314MeV and 4.012MeV, respectively, then the kinetic energy of ${}^{17}_{8}O$ is ______MeV. (Rounded off to one decimal place)

(Masses are given in units of u = 931.5MeV/c^2)

Q.55 A satellite of mass 10kg, in a circular orbit around a planet, is having a speed v=200m/s. The total energy of the satellite is _______kJ. (Rounded off to nearest integer)

Q.56 When a system of multiple long narrow slits (width 2μm and period 4μm) is illuminated with a laser of wavelength 600nm. There are 40 minima between the two consecutive principal maxima observed in its diffraction pattern. Then maximum resolving power of the system is _______.

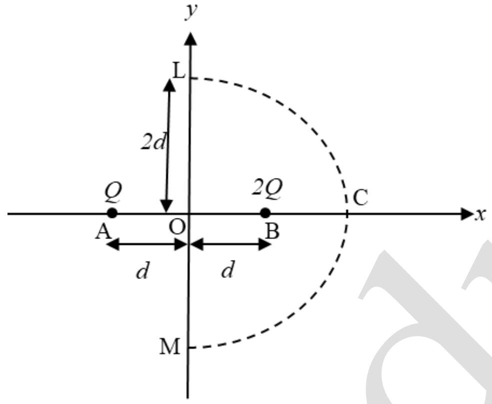
Q.57 Consider a thick biconvex lens (thickness t = 4cm and refractive index n = 1.5) whose magnitudes of the radii of curvature R_1 and R_2, of the first and second surfaces are 30cm and 20cm, respectively. Surface 2 is silvered to act as a mirror. A point object is placed at point A on the axis (OA = 60cm) as shown in the figure. If its image is formed at point Q, the distance d between O and Q is ______cm. (Rounded off to two decimal places)



Q.58 An unstable particle created at a point P moves with a constant speed of $0.998c$ until it decays at a point Q. If the lifetime of the particle in its rest frame is 632ns, the distance between points P and Q is ______m. (Rounded off to the nearest integer)

($c = 3 \times 10^8$ m/s)

Q.59 Two positive charges Q and $2Q$ are kept at points A and B, separated by a distance $2d$, as shown in the figure. MCL is a semicircle of radius $2d$ centered at the origin O. If Q = 2C and d = 10cm, the value of the line integral $\int_M^L \vec{E}.\overrightarrow{dl}$ (where $\vec{E}$ represents electric field) along the path MCL will be _________V.



Q.60 A time dependent magnetic field inside a long solenoid of radius 0.05m is given by $\vec{B}(t) = B_0 \sin \omega t\, \hat{z}$. If ω=100rad/s and B_0= 0.98Weber/m^2, then the amplitude of the induced electric field at a distance of 0.07m from the axis of the solenoid is ______V/m. (Rounded off to two decimal places)